Practical Stabilization of Driftless Homogeneous Systems Based on the Use of Transverse Periodic Functions

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Abstract: We address the problem of practical stabilization of driftless nonlinear control systems with homogeneous vector fields. A general feedback design approach, based on the concept of transverse functions recently introduced by the authors, is presented. This approach allows to achieve global stabilization of any neighborhood of the origin, possibly in the presence of additive —known or measured— perturbations acting on the system.

Key-words: practical stabilization, nonlinear system, homogeneous vector field, transverse function

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Stabilisation pratique de systèmes homogènes sans dérive, à partir de fonctions transverses périodiques

Résumé : Ce rapport concerne la stabilisation pratique de systèmes nonlinéaires sans dérive dont les champs de vecteurs sont homogènes. Nous présentons une méthode générale de synthèse de lois de commande, basée sur la notion de fonction transverse introduite dans un précédent rapport. Cette méthode permet de rendre globalement stable un voisinage arbitrairement petit de l'origine, même lorsque des perturbations additives —connues ou mesurées— agissent sur le système.

Mots-clés : stabilisation pratique, système nonlinéaire, champ de vecteur homogène, fonction transverse

Contents

1 Introduction

Asymptotic stabilization, by means of a continuous feedback control, of the origin of a smooth controllable system

$$\mathcal{S}: \qquad \dot{x} = \sum_{i=1}^{m} b_i(x)u_i \tag{1}$$

with a number m of control inputs smaller than the system's dimension n, is a non-trivial problem, for which several solutions have been proposed during the last decade. These solutions, whether or not explicitly based on the introduction of the time variable in the control law —recall that pure-state feedback stabilization is impossible when the vectors $b_1(0)\ldots, b_m(0)$ are independent [2]— involve a dynamic extension of $\mathcal{S}$. For instance, this extension may be of the form $\dot{z}_1 = \omega\dot{z}_2$, $\dot{z}_2 = -\omega\dot{z}_1$ in the case of a periodic stabilizer. It provides the excitation needed to asymptotically stabilize the origin of the initial state vector. The design of an asymptotic stabilizer for $\mathcal{S}$, as a function of the augmented state vector, has been handled by applying, and often combining, various techniques (time-averaging, center manifold, backstepping,...). By comparison with linear control design techniques, tools to finely tune the control performance have seldom been developed.

Another matter of practical concern is the robustness of asymptotic stabilizers of $\mathcal{S}$. For instance, it is shown in [9] that, with homogeneous exponential stabilizers, the slightest imprecision in the modelling of one of the control vector fields —v.f.— of the system may result in the loss of stability. To correct this lack of robustness with respect to modelling errors, *hybrid* open-loop/feedback control laws can be used [1, 11]. The problem is, however, only displaced in this way, since such hybrid stabilizers are not, for instance, robust to small variations of the sampling time period. As for Lipschitz stabilizers, they may be robust to both modelling errors [10] and sampling frequency variations, but their practical usefulness is hampered by the fact that they yield a very slow rate of convergence —polynomial instead of exponential.

The above considerations call for a distinct way of approaching the control of $\mathcal{S}$. In this paper, we consider the problem of asymptotic stabilization of an arbitrary small *neighborhood of the origin* , with the extra requirement of *robustness to known (or measured)*, but otherwise arbitrary, *additive perturbations* acting on the system. More precisely, if $b_0(x,t)$ is a term added to the right-hand side of $\mathcal{S}$, the control solution should allow to reject this term in order to still achieve asymptotic stabilization of an arbitrary small neighborhood of the origin. The present paper develops a solution to this problem when $\mathcal{S}$ is homogeneous and controllable. It is a generalization of the solution described in [13] for chained form systems, and it is based on a theorem [12] which provides a characterization of the Lie Algebra Rank Condition in terms of the existence of bounded periodic functions which are *transverse* to the distribution generated by a set of smooth v.f. With respect to control design techniques discussed above, the proposed solution also involves a dynamic extension of the system. However, it has a few distinctive features, such as the explicit use of transverse functions, as defined in [12], and the possibility of interpreting the time-derivatives of the exogenous dynamic variables as frequency control inputs. This interpretation can be traced back to

[3], a work which has inspired us the controllability characterization in [12]. An interest of considering the class of homogeneous systems is that they allow for global stabilization results. Another one is that every smooth driftless system $\mathcal{S}$, locally controllable, can be locally approximated by a controllable homogeneous system [15, 5].

The paper is organized as follows. In Section 2 we recall some definitions and state a few technical results. The main result is given in Section 3. The design algorithm is summarized in in Section 4, and illustrated by an example.

The following notation is used. *Smooth* means of class $\mathcal{C}^\infty$. $B_n(0,\epsilon)$ denotes the closed ball in $\mathbb{R}^n$ centered at zero, and of radius ϵ. $\mathbb{T} \triangleq \mathbb{R}/2\pi\mathbb{Z}$ denotes the one-dimensional torus. $I_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix. $L_b f$ denotes the Lie derivative of the function f along the v.f. b.

2 Preliminary definitions and results

2.1 Homogeneity

We recall below a few basic definitions (see e.g. [5, 6] for more details).
Given a *weight vector* $r = (r_1, \ldots, r_n)$ $(r_i > 0\ \forall i)$, a *dilation* Δ^r_λ $(\lambda > 0)$ on $\mathbb{R}^n$ is a map from $\mathbb{R}^n$ to $\mathbb{R}^n$ defined by $\Delta^r_\lambda x \triangleq (\lambda^{r_1}x_1, \ldots, \lambda^{r_n}x_n)$. In this paper, we shall only consider dilations with integer-valued weights, i.e. $r_i \in \mathbb{N}\ \forall i$. This is not a limitation because for smooth functions —or v.f.—, it is always possible to rescale the weight vector so that this assumptions be satisfied.
A function $f \in \mathcal{C}^0(\mathbb{R}^n;\mathbb{R})$ is *homogeneous of degree l with respect to the family of dilations* $(\Delta^r_\lambda)_{\lambda>0}$, or more concisely *$\Delta^r$-homogeneous of degree l*, if $\forall\lambda > 0,\ \forall x \in \mathbb{R}^n,\ f(\Delta^r_\lambda x) = \lambda^l f(x)$.
A *Δ^r-homogeneous norm* is a positive definite function on $\mathbb{R}^n$, Δ^r-homogeneous of degree one.
A smooth v.f. X on $\mathbb{R}^n$ is *Δ^r-homogeneous of degree d* if, for all $i = 1, \ldots, n$, the function $x \longmapsto X_i(x)$ is Δ^r-homogeneous of degree $d + r_i$. Let us recall that if X_1, X_2 are Δ^r-homogeneous of degree d_1, d_2 respectively, then $[X_1, X_2]$ is Δ^r-homogeneous of degree d_1+d_2. We remind the reader that, smooth and homogeneous functions, or v.f., are polynomial.

Finally, we say that a set $\{b_1, \ldots, b_m\}$ of v.f. —or the associated system $\mathcal{S}$— is *nilpotent of order* d+1 if Lie brackets of these v.f. of length larger than, or equal to, d+1 are identically zero.

2.2 Fröbenius Theorem in the homogeneous case

The purpose of this section is merely to particularize the consecrated theorem of Fröbenius to the case of homogeneous v.f., and point out extra global properties resulting from this adaptation. The proof is given in the appendix.

Theorem 1 *Let $b_1, \ldots, b_m$ denote smooth v.f. on $\mathbb{R}^n$, Δ^r-homogeneous for some weight vector r. Assume that, in a neighborhood of $x = 0$, the distribution generated by $b_1, \ldots, b_m$ is involutive of dimension m.*

Then, there exist a set $I = \{i_1, \ldots, i_{n-m}\}$ of indices in $\{1, \ldots, n\}$ and a mapping $\phi \in C^\infty(\mathbb{R}^n; \mathbb{R}^{n-m})$ which satisfy the three following properties:

1. $\forall i \in \{1, \ldots, m\}, \forall j \in \{1, \ldots, n-m\} \qquad L_{b_i}\phi_j \equiv 0$
2. *For any $j = 1, \ldots, n-m$, ϕ_j is Δ^r-homogeneous of degree r_{i_j}.*
3. *The matrix $\frac{\partial \phi}{\partial \bar{x}}(x)$ is invertible at any x, and $\frac{\partial \phi}{\partial \bar{x}}(0) = I_{n-m}$, with $\bar{x} = (x_{i_1}, \ldots, x_{i_{n-m}})$.*

2.3 Free systems

We refer the reader to [8, 16] for a comprehensive introduction.

Let us consider a finite set of indeterminates $X_1, \ldots, X_m$, and denote by Lie(X) the free Lie algebra over $\mathbb{R}$ generated by the X_i's. We also denote by $\mathcal{F}(X)$ the set of formal brackets in the X_i's. We use the notations $\ell(B)$ to denote the length of a formal bracket B, and $\lambda(B), \rho(B)$ to denote its left and right factor, i.e. for $\ell(B) > 1$, $B = [\lambda(B), \rho(B)]$. This latter notation extends naturally to $\lambda\rho(B), \rho^2(B), \ldots$ and so forth as far as the length of B permits it. For any set $\mathbf{b} \triangleq \{b_1, \ldots, b_m\}$ of smooth v.f., and any $B \in \mathcal{F}(X)$, we denote by $\mathrm{Ev}_{\mathbf{b}}(B)$ the *evaluation map* , i.e. $\mathrm{Ev}_{\mathbf{b}}(X_i) = b_i$, and $\mathrm{Ev}_{\mathbf{b}}([\lambda(B), \rho(B)]) = [\mathrm{Ev}_{\mathbf{b}}(\lambda(B)), \mathrm{Ev}_{\mathbf{b}}(\rho(B))]$.

The definition of a P. Hall basis of Lie(X) —in the original, narrow sense— is now recalled.

Definition 1 *A P. Hall basis $\mathcal{B}$ of Lie(X) is a totally ordered subset of $\mathcal{F}(X)$ such that*

1. *Each X_i belongs to $\mathcal{B}$.*
2. *If $B, B' \in \mathcal{B}$ have length $\ell(B) < \ell(B')$, then $B < B'$.*
3. *If $\lambda(B), \rho(B) \in \mathcal{F}(X)$ then, $B = [\lambda(B), \rho(B)] \in \mathcal{F}(X)$ if and only if $\lambda(B), \rho(B) \in \mathcal{B}$ with $\lambda(B) < \rho(B)$, and either (i) $\rho(B)$ is one of the X_i's or (ii) $\rho(B) = [\lambda\rho(B), \rho^2(B)]$ with $\lambda\rho(B) \leq \lambda(B)$.*

Let $\mathcal{B} = \{B_1, \ldots, B_k, \ldots\}$ denote a P. Hall basis, and $\mathcal{B}_{\bar{n}} = \{B_1, \ldots, B_{\bar{n}}\}$ be the subset composed of the first $\bar{n}$ elements of $\mathcal{B}$ for some arbitrary integer $\bar{n}$. One can associate the following *free system* [8] with $\mathcal{B}_{\bar{n}}$:

$$\mathcal{F}_{\bar{n}} : \left\{ \begin{array}{lcll} \dot{x}_B & = & u_i & \text{for } B = X_i \\ \dot{x}_B & = & x_{\lambda(B)}\dot{x}_{\rho(B)} & \text{for } B \in \mathcal{B}_{\bar{n}} \text{ with } \ell(B) > 1\,. \end{array} \right. \tag{2}$$

It is straightforward to verify that (2) defines a driftless system on $\mathbb{R}^{\bar{n}}$ of the form $\mathcal{S}$. Following [7], we have indexed the state variables by elements of the P. Hall basis. This notation is motivated by the following result [7].

Lemma 1 *Let* $\bar{\mathbf{b}} \triangleq \{\bar{b}_1, \ldots, \bar{b}_m\}$ *denote the set of control v.f. associated with the free system* $\mathcal{F}_{\bar{n}}$*. Then,*

1. *For any* $B \in \mathcal{B}_{\bar{n}}$*,*
$$Ev_{\bar{\mathbf{b}}}(B) = a_B \frac{\partial}{\partial x_B} + \sum_{B<B'} c_{B,B'} \frac{\partial}{\partial x_{B'}}$$
with a_B *a non zero constant, so that* $\mathcal{F}_{\bar{n}}$ *satisfies the Lie Algebra Rank Condition on* $\mathbb{R}^{\bar{n}}$*.*

2. *For any set of strictly negative integers* $d_1, \ldots, d_m$*, each v.f.* $\bar{b}_i$ *is* Δ^r*-homogeneous of degree* d_i *with* r *the weight vector defined by*
$$\begin{cases} r_B = -d_i & \text{for } B = X_i \\ r_B = r_{\lambda(B)} + r_{\rho(B)} & \text{for } \ell(B) > 1\,. \end{cases}$$

From now on, we shall only consider subsets $\mathcal{B}_{\bar{n}}$ of a specific P. Hall basis $\mathcal{B}$ obtained by specifying an order on brackets which have equal lengths.

Specific order:

$$\begin{cases} X_i < X_j \Longleftrightarrow i < j \\ \text{If } \ell(B) = \ell(B') > 1,\ \Big(B < B'\Big) \Longleftrightarrow \Big(\big(\lambda(B) < \lambda(B')\big) \text{ or } \big(\lambda(B) = \lambda(B') \text{ and } \rho(B) < \rho(B')\big)\Big) \end{cases}$$

Lemma 2 *Consider a smooth controllable system* $\mathcal{S}$ *on* $\mathbb{R}^n$*, such that the v.f.* $b_1, \ldots, b_m$ *are* Δ^r*-homogeneous of degree* $d_1, \ldots, d_m < 0$ *respectively, for some weight vector* r*.*

Then, there exists $\bar{n} \in \mathbb{N}$*, and a controllable and homogeneous dynamic extension of* $\mathcal{S}$ *on* $\mathbb{R}^{\bar{n}}$*:*

$$\mathcal{S}^e : \quad \begin{cases} \dot{x} = \sum_{i=1}^{m} b_i(x) u_i \\ \dot{y} = \sum_{i=1}^{m} g_i(x,y) u_i \end{cases} \tag{3}$$

which is equivalent, via a global change of coordinates, to the free system $\mathcal{F}_{\bar{n}}$*. Furthermore, if* $d+1$ *denotes the order of nilpotency of* $\{b_1, \ldots, b_m\}$*,* $\bar{n}$ *can be chosen as the number of brackets in* $\mathcal{B}$ *of length at most equal to* d*.*

This result, first proved in [14], is known as the Rotschild-Stein lifting theorem —see also [4]— for further developments. For completeness, a proof based on Theorem 1 is given in the appendix.

2.4 Transverse functions

The following result was proved in [12].

Proposition 1 *Let $\bar{b}_1, \ldots, \bar{b}_m$ denote the v.f. of $\mathcal{F}_{\bar{n}}$. Then, there exists a family $(f_\epsilon)_{\epsilon>0}$ of functions in $\mathcal{C}^\infty(\mathbb{T}^{\bar{n}-m}; B_{\bar{n}}(0,\epsilon))$ such that*

$$\forall \epsilon > 0\,,\ \forall \theta \in \mathbb{T}^{\bar{n}-m}\,, \qquad Det(H_\epsilon(\theta)) \neq 0\,, \tag{4}$$

with

$$H_\epsilon(\theta) = \left(\bar{b}_1(f_\epsilon(\theta)) \ \ldots\ \bar{b}_m(f_\epsilon(\theta)) \ -\frac{\partial f_\epsilon}{\partial \theta_1}(\theta) \ \ldots\ -\frac{\partial f_\epsilon}{\partial \theta_{\bar{n}-m}}(\theta)\right) \tag{5}$$

2.5 A useful transformation

The next result points out the existence of a mapping whose role is central to the stabilization technique proposed in the next section. The proof is given in the appendix.

Lemma 3 *Let $\bar{b}_1, \ldots, \bar{b}_{\bar{n}}$ denote smooth v.f. on $\mathbb{R}^{\bar{n}}$ such that*

a) for some weight vector $\bar{r}$, each $\bar{b}_i$ is $\Delta^{\bar{r}}$-homogeneous of negative degree,

b) $\bar{b}_1(0), \ldots, \bar{b}_{\bar{n}}(0)$ are linearly independent,

c) $\{\bar{b}_1, \ldots, \bar{b}_{\bar{n}}\}$ is a basis, over $\mathbb{R}$, of $Lie\{\bar{b}_1, \ldots, \bar{b}_{\bar{n}}\}$.

Then, there exists a mapping $\phi \in C^\infty(\mathbb{R}^{\bar{n}} \times \mathbb{R}^{\bar{n}}; \mathbb{R}^{\bar{n}})$ such that

1. $\forall (x,y) \in \mathbb{R}^{\bar{n}} \times \mathbb{R}^{\bar{n}}$, $\forall \lambda \geq 0 \quad \phi(\Delta^{\bar{r}}_\lambda x, \Delta^{\bar{r}}_\lambda y) = \Delta^{\bar{r}}_\lambda \phi(x,y)$,
2. $\forall i \in \{1, \ldots, \bar{n}\}$, $\forall (x,y) \in \mathbb{R}^{\bar{n}} \times \mathbb{R}^{\bar{n}}$,
$$\frac{\partial \phi}{\partial x}(x,y)\bar{b}_i(x) + \frac{\partial \phi}{\partial y}(x,y)[\bar{b}_i(x) - \bar{b}_i(x-y)] = 0\ , \tag{6}$$
3. $\forall (x,y) \in \mathbb{R}^{\bar{n}} \times \mathbb{R}^{\bar{n}}$, $\frac{\partial \phi}{\partial y}(x,y)$ *is invertible and* $\frac{\partial \phi}{\partial y}(0,0) = I_{\bar{n}}$,
4. $\phi(x,y) = 0 \iff y = 0$.

3 Practical stabilization of driftless homogeneous systems

The proposed control approach is based on the following proposition, which is the main result of the paper.

Proposition 2 *Consider a smooth controllable system $\mathcal{S}$ on $\mathbb{R}^n$, such that the v.f. $b_1, \ldots, b_m$ are Δ^r-homogeneous of degree $d_1, \ldots, d_m < 0$ respectively, for some weight vector r. Let*

$$\mathcal{S}^e : \qquad \dot{\bar{x}} = \sum_{i=1}^{m} \bar{b}_i(\bar{x}) u_i \tag{7}$$

denote a dynamic extension of $\mathcal{S}$ on $\mathbb{R}^{\bar{n}}$ equivalent to the free system $\mathcal{F}_{\bar{n}}$, as defined in Lemma 2. Denote, for $i = 1, \ldots, \bar{n}$, $\bar{b}_i = Ev_{\bar{\mathbf{b}}}(B_i)$ with $\bar{\mathbf{b}} = \{\bar{b}_1, \ldots, \bar{b}_m\}$.

Then, there exists $\phi \in C^\infty(\mathbb{R}^{\bar{n}} \times \mathbb{R}^{\bar{n}}; \mathbb{R}^{\bar{n}})$ which satisfies Properties 1–4 of Lemma 3. Furthermore, for any function $f_\epsilon \in C^\infty(\mathbb{T}^{\bar{n}-m}; B_{\bar{n}}(0, \epsilon))$, any function $\theta \in C^\infty(\mathbb{R}; \mathbb{T}^{\bar{n}-m})$, and along any trajectory of $\mathcal{S}^e$,

$$\frac{d}{dt}(\phi(\bar{x}, \bar{x} - f_\epsilon(\theta))) = \frac{\partial \phi}{\partial y}(\bar{x}, \bar{x} - f_\epsilon(\theta)) H_\epsilon(\theta) U \tag{8}$$

with

$$\begin{aligned} U &\triangleq \left(u_1, \ldots, u_m, \dot{\theta}_1, \ldots, \dot{\theta}_{\bar{n}-m}\right)^T \\ H_\epsilon(\theta) &\triangleq \left(\bar{b}_1(f_\epsilon(\theta)) \; \ldots \; \bar{b}_m(f_\epsilon(\theta)) \; -\frac{\partial f_\epsilon}{\partial \theta_1}(\theta) \; \ldots \; -\frac{\partial f_\epsilon}{\partial \theta_{\bar{n}-m}}(\theta)\right) . \end{aligned} \tag{9}$$

Proof: By application of Lemma 2, there exists a dynamic extension $\mathcal{S}^e$ of $\mathcal{S}$ equivalent to a free system $\mathcal{F}_{\bar{n}}$. From Lemma 1, the v.f. $\bar{b}_1, \ldots, \bar{b}_{\bar{n}}$ satisfy all assumptions of Lemma 3. The existence of ϕ follows by application of this lemma. As for relation (8), it is easily obtained by differentiating $\phi(\bar{x}, \bar{x} - f_\epsilon(\theta))$ with respect to time and by using (6) with $y = \bar{x} - f_\epsilon(\theta)$ in the calculations. ∎

Corollary 1 *For any Hurwitz-stable matrix $K \in \mathbb{R}^{\bar{n} \times \bar{n}}$, and any transverse function f_ϵ —i.e. such that the matrix $H_\epsilon(\theta)$ is invertible for any θ— the smooth control law*

$$U_\epsilon(\bar{x}, \theta) \triangleq H_\epsilon^{-1}(\theta) \left(\frac{\partial \phi}{\partial y}(\bar{x}, \bar{x} - f_\epsilon(\theta))\right)^{-1} K \phi(\bar{x}, \bar{x} - f_\epsilon(\theta)) \tag{10}$$

ensures exponential stabilization of $\phi = 0$, and ultimate boundedness of $|x|$ by ϵ.

Proof: It directly follows from (8) and (10) that along the trajectories of the closed loop system

$$\dot{\phi} = K\phi$$

so that, since K is Hurwitz by definition, ϕ converges exponentially to zero. From Properties 1 and 4 in Lemma 3, one can show that $\phi(\bar{x}, \bar{x} - f) \longrightarrow 0$ implies $\bar{x} - f \longrightarrow 0$ when f is restricted to a bounded set. This implies ultimate boundedness of $|\bar{x}|$ by ϵ and, since x is a sub-vector of $\bar{x}$, ultimate boundedness of $|x|$ by ϵ also. ∎

Remark 1 In view of the control expression (10), the convergence of ϕ to zero implies that the control tends to zero. This means in particular that $\dot{\theta}$, interpreted as a vector of self-adapting control frequencies, converges to zero. Note also that the closed-loop equation $\dot{\phi} = K\phi$ can still be enforced when $\mathcal{S}$ is perturbed by a known (or measured), but otherwise arbitrary, additive v.f. b_0. Indeed, it suffices in this case to add the adequate compensating term to the expression of the extended control U. This is left as an exercise to the interested reader.

4 Design algorithm and example

In view of the previous results, we can propose the following control design algorithm for the practical stabilization of an arbitrary controllable homogeneous system $\mathcal{S}$.

Control design :

Step 1: Find a dynamic extension $\mathcal{S}^e$ of $\mathcal{S}$ equivalent to a free system $\mathcal{F}_{\bar{n}}$. First, $\bar{n}$ has to be determined. If $d+1$ denotes the order of nilpotency of $\mathcal{S}$, $\bar{n}$ can always be chosen as the number of elements de $\mathcal{B}$ of length d at most. Then, with x denoting the state vector of $\mathcal{F}_{\bar{n}}$, $\mathcal{S}^e$ is just $\mathcal{F}_{\bar{n}}$ written in the coordinates $\bar{x} = \bar{\psi}(x)$ defined by

$$\bar{\psi}(x) \triangleq \begin{pmatrix} \psi(x) \\ (0 \;\; I_{\bar{n}-n})Px \end{pmatrix}$$

with ψ the solution of

$$\frac{\partial \psi}{\partial x}(x) = \begin{pmatrix} b_1(x) & \dots & b_{\bar{n}}(x) \end{pmatrix} \begin{pmatrix} \bar{b}_1(x) & \dots & \bar{b}_{\bar{n}}(x) \end{pmatrix}^{-1} \qquad \frac{\partial \psi}{\partial x}(0) = 0$$

where $b_i = \mathrm{Ev}_{\mathbf{b}}(B_i)$ and $\bar{b}_i = \mathrm{Ev}_{\bar{\mathbf{b}}}(B_i)$ —$\mathbf{b}$ denoting the set of control v.f of $\mathcal{S}$, $\bar{\mathbf{b}}$ the set of control v.f of $\mathcal{F}_{\bar{n}}$, and B_i an element of $\mathcal{B}_{\bar{n}}$. The matrix P is any permutation matrix such that

$$\frac{\partial \psi}{\partial x}(0)P^T \begin{pmatrix} I_n \\ 0 \end{pmatrix}$$

is invertible.

Step 2: Compute a family $(f_\epsilon)_{\epsilon>0}$ of transverse functions for $\mathcal{S}^e$. A design algorithm is proposed in [12] to compute transverse functions $\bar{f}_\epsilon$ for any free system $\mathcal{F}_{\bar{n}}$. Transverse functions for $\mathcal{S}^e$ are then obtained by setting $f_\epsilon = \bar{\psi} \circ \bar{f}_\epsilon$.

Step 3: Find a mapping ϕ which satisfies the conditions of Lemma 3 for the v.f. $\bar{b}_1, \dots, \bar{b}_{\bar{n}}$ associated with $\mathcal{S}^e$.

Step 4: Form the control equation in $\phi(\bar{x}, \bar{x} - f_\epsilon(\theta))$ and determine a feedback control which exponentially stabilizes ϕ to zero —for example, apply the control law (10).

Before illustrating this algorithm with an example, let us mention that the case of chained form systems has previously been treated in [13]. For these systems, a result stronger than what the direct application of the previous algorithm yields was obtained. More precisely, the algorithm involves a dynamic extension of the chained system which, in fact, is not needed, due to specific structural properties of chained systems. Conditions under which the dimension of the dynamic extension can be chosen smaller than the dimension $\bar{n}$ of a free system are currently investigated by the authors.

We consider the following example on $\mathbb{R}^3$:

$$\left\{\begin{array}{lcl} \dot{x}_1 & = & u_1 \\ \dot{x}_2 & = & u_2 \\ \dot{x}_3 & = & x_1^2 u_2 \end{array}\right. \tag{11}$$

whose control v.f. are both Δ^r-homogeneous of degree -1, with $r = (1,1,3)^T$. It is simple to verify that this system is nilpotent of order 4.

Step 1: The dynamic extension of (11)

$$\left\{\begin{array}{lcl} \dot{\bar{x}}_1 & = & u_1 \\ \dot{\bar{x}}_2 & = & u_2 \\ \dot{\bar{x}}_3 & = & \bar{x}_1^2 u_2 \\ \dot{\bar{x}}_4 & = & \bar{x}_1 u_2 \end{array}\right. \tag{12}$$

is clearly equivalent to the free system $\mathcal{F}_4$ associated with

$$\mathcal{B}_4 = \{X_1, X_2, [X_1, X_2], [X_1, [X_1, X_2]]\}\,.$$

The change of coordinates ψ from $\mathcal{F}_4$ to (12) is just a permutation, i.e.

$$\psi^{-1}(\bar{x}) = (\bar{x}_1, \bar{x}_2, \bar{x}_4, \bar{x}_3)^T\,.$$

Step 2: An example of transverse function for the free system $\mathcal{F}_4$ was computed in [12]. Under the change of coordinates ψ, this yields the following transverse function for (12):

$$f(\theta_1,\theta_2) = \begin{pmatrix} \sin\theta_1 + \eta\sin\theta_2 \\ \cos\theta_1 \\ \frac{\eta^3}{2}\sin 2\theta_2 + \frac{\eta}{2}\sin\theta_2\sin 2\theta_1 + \eta^2\sin^2\theta_2\cos\theta_1 \\ \frac{1}{4}\sin 2\theta_1 + \eta^2\cos\theta_2 + \eta\sin\theta_2\cos\theta_1 \end{pmatrix}$$

with η a parameter which has to be taken large enough —for example, $\eta \geq 5/2$. A family $(f_\epsilon)_{\epsilon>0}$ of transverse functions is simply given by $(\Delta^{\bar{r}}_{\mu(\epsilon)} f)_{\epsilon>0}$, with $\bar{r} = (1,1,3,2)$ and $\mu(\epsilon)$ a number chosen small enough so that $\Delta^{\bar{r}}_{\mu(\epsilon)} f$ ranges within the ball of radius ϵ.

Step 3: A function ϕ for which all conditions of Lemma 3 are verified for the v.f. of (12) is given by

$$\phi(\bar{x},\bar{y}) = (\bar{y}_1, \bar{y}_2, \bar{y}_3 - 2\bar{x}_4\bar{y}_1 + \bar{x}_2\bar{y}_1^2, \bar{y}_4 - \bar{x}_2\bar{y}_1)^T$$

Step 4: This step is left to the interested reader.

Appendix

Proof of Theorem 1

From the Fröbenius theorem, there exists a neighborhhod $\mathcal{U}$ of the origin and a function $\bar{\phi} \in \mathcal{C}^\infty(\mathcal{U};\mathbb{R}^{n-m})$ such that Property 1 is satisfied for $\bar{\phi}$, and the matrix

$$D \triangleq \frac{\partial \bar{\phi}}{\partial \bar{x}}(0)$$

is invertible, with $\bar{x} = (x_{i_1}, \dots, x_{i_{n-m}})$ and $\{i_1, \dots, i_{n-m}\}$ a set of indices in $\{1, \dots, n\}$. Without loss of generality, we can assume that $D = I_{n-m}$ —possibly after changing $\bar{\phi}$ into $D^{-1}\bar{\phi}$. We define ϕ by

$$\phi(x) = \frac{1}{q!}\frac{\partial^q}{\partial \lambda^q}\left(\lambda^q \Delta^{\bar{r}}_{\lambda^{-1}}\,\bar{\phi}(\Delta^r_\lambda x)\right)_{|\lambda=0}$$

with $\bar{r} = (r_{i_1}, \dots, r_{i_{n-m}})$, and $q \in \mathbb{N} \geq \max\{r_1, \dots, r_n\}$. We show the three properties of Theorem 1.

Property 1 : We have

$$\begin{aligned}
\frac{\partial \phi}{\partial x}(x) b_i(x) &= \frac{1}{q!}\frac{\partial^q}{\partial \lambda^q}\left(\lambda^q \Delta^{\bar{r}}_{\lambda^{-1}}\frac{\partial \bar{\phi}}{\partial x}(\Delta^r_\lambda x)\Delta^r_\lambda b_i(x)\right)_{|\lambda=0} \\
&= \frac{1}{q!}\frac{\partial^q}{\partial \lambda^q}\left(\lambda^q \Delta^{\bar{r}}_{\lambda^{-1}}\frac{\partial \bar{\phi}}{\partial x}(\Delta^r_\lambda x) b_i(\Delta^r_\lambda x)\lambda^{-d_i}\right)_{|\lambda=0} \\
&= 0
\end{aligned} \tag{13}$$

where d_i denotes the degree of homogeneity of b_i, i.e. $b_i(\Delta^r_\lambda x) = \Delta^r_\lambda b_i(x)\lambda^{d_i}$.

Property 2 : We have

$$\begin{aligned}
\phi(\Delta_\beta x) &= \frac{1}{q!}\frac{\partial^q}{\partial \lambda^q}\left(\lambda^q \Delta^{\bar{r}}_{\lambda^{-1}}\bar{\phi}(\Delta^r_{\lambda\beta} x)\right)_{|\lambda=0} \\
&= \frac{1}{\beta^q}\frac{1}{q!}\Delta^{\bar{r}}_\beta\frac{\partial^q}{\partial \lambda^q}\left((\lambda\beta)^q \Delta^{\bar{r}}_{(\lambda\beta)^{-1}}\bar{\phi}(\Delta^r_{\lambda\beta} x)\right)_{|\lambda=0} \\
&= \frac{1}{q!}\Delta^{\bar{r}}_\beta\frac{\partial^q}{\partial \gamma^q}\left(\gamma^q \Delta^{\bar{r}}_{\gamma^{-1}}\bar{\phi}(\Delta^r_\gamma x)\right)_{|\gamma=0} \\
&= \Delta^{\bar{r}}_\beta \phi(x)
\end{aligned}$$

with the third equality coming from the fact that, for any smooth function f on $\mathbb{R}$,

$$\frac{\partial^k}{\partial \lambda^k} f(\lambda\beta)_{\lambda=0} = \beta^k \frac{\partial^k}{\partial \lambda^k} f(\lambda)_{\lambda=0}\,.$$

Property 3 : We have

$$\begin{aligned}\frac{\partial\phi}{\partial\bar{x}}(0) &= \frac{\partial}{\partial\bar{x}}\left(\frac{1}{q!}\frac{\partial^q}{\partial\lambda^q}(\lambda^q\Delta_{\lambda^{-1}}^{\bar{r}}\bar{\phi}(\Delta_\lambda^r x))_{|\lambda=0}\right)_{|x=0} \\ &= \frac{1}{q!}\frac{\partial^q}{\partial\lambda^q}\left(\lambda^q\Delta_{\lambda^{-1}}^{\bar{r}}\frac{\partial\bar{\phi}(\Delta_\lambda^r\circ\Delta_\lambda)}{\partial\bar{x}}(x)_{|x=0}\right)_{|\lambda=0} \\ &= \frac{1}{q!}\frac{\partial^q}{\partial\lambda^q}\left(\lambda^q\Delta_{\lambda^{-1}}^{\bar{r}}\frac{\partial\bar{\phi}}{\partial\bar{x}}(\Delta_\lambda^r x)_{|x=0}\Delta_\lambda^{\bar{r}}\right)_{|\lambda=0} \\ &= \frac{1}{q!}\frac{\partial^q}{\partial\lambda^q}(\lambda^q I_{n-m})_{|\lambda=0} = I_{n-m}\,,\end{aligned}$$

where the third equality comes from the fact that

$$\frac{\partial\bar{\phi}}{\partial x}(\Delta_\lambda^r x)\frac{\partial}{\partial\bar{x}}\Delta_\lambda^r x = \frac{\partial\bar{\phi}}{\partial\bar{x}}(\Delta_\lambda^r x)\Delta_\lambda^{\bar{r}}\,. \tag{14}$$

By continuity, $\frac{\partial\phi}{\partial\bar{x}}(x)$ is also invertible in a neighborhood of the origin. Invertibility on $\mathbb{R}^n$ is then obtained from (14), after differentiating with respect to $\bar{x}$ the equality $\phi(\Delta_\lambda^r x) = \Delta_\lambda^{\bar{r}}\phi(x)$. ∎

Proof of Lemma 2

Let $d+1$ denote the order of nilpotency of $\mathcal{S}$. We take $\bar{n}$ as the number of elements of $\mathcal{B}$ of length d at most. We denote $b_i = \mathrm{Ev}_{\mathbf{b}}(B_i)$ and $\bar{b}_i = \mathrm{Ev}_{\bar{\mathbf{b}}}(B_i)$, for $i = 1,\ldots,n$, where $\mathbf{b} = \{b_1,\ldots,b_m\}$ denotes the v.f of $\mathcal{S}$, and $\bar{\mathbf{b}} = \{\bar{b}_1,\ldots,\bar{b}_m\}$ denotes the v.f of $\mathcal{F}_{\bar{n}}$. Consider now the following distribution at an arbitrary point $(x,z)\in\mathbb{R}^{n+\bar{n}}$:

$$\mathrm{Span}\left\{\begin{pmatrix} b_1(x) \\ \bar{b}_1(z)\end{pmatrix},\ldots,\begin{pmatrix} b_{\bar{n}}(x) \\ \bar{b}_{\bar{n}}(z)\end{pmatrix}\right\}.$$

By Property 1 of Lemma 1 and Property 2 of Lemma 2, its dimension is $\bar{n}$ and it is involutive. Define the following matrices $B(x) \triangleq (b_1(x)\ \ldots\ b_{\bar{n}}(x))$ and $\bar{B}(z) \triangleq (\bar{b}_1(z)\ \ldots\ \bar{b}_{\bar{n}}(z))$. From Property 2 of Lemma 1, and by the assumption of Lemma 2, each v.f.

$$\begin{pmatrix} x \\ z\end{pmatrix} \longmapsto \begin{pmatrix} b_i(x) \\ \bar{b}_i(z)\end{pmatrix}$$

is $\Delta^{\tilde{r}}$-homogeneous with $\tilde{r} = (\bar{r}, r)$ and where $\bar{r}$ (resp. r) denotes the weight vector associated with $\mathcal{F}_{\bar{n}}$ (resp. $\mathcal{S}$). By Theorem 1, there exists a mapping $\phi\in C^\infty(\mathbb{R}^{n+\bar{n}};\mathbb{R}^n)$ such that

$$\forall (x,z)\in\mathbb{R}^{n+\bar{n}}\,,\quad \frac{\partial\phi}{\partial x}(x,z)B(x) + \frac{\partial\phi}{\partial z}(x,z)\bar{B}(z) = 0 \tag{15}$$

Therefore

$$\forall (x,z)\in\mathbb{R}^{n+\bar{n}}\,,\quad \frac{\partial\phi}{\partial z}(x,z) = -\frac{\partial\phi}{\partial x}(x,z)B(x)\bar{B}(z)^{-1} \tag{16}$$

and

$$\left(\frac{\partial\phi}{\partial x}(x,z)\ \ \frac{\partial\phi}{\partial z}(x,z)\right) = \frac{\partial\phi}{\partial x}(x,z)\ \left(I_n \quad -B(x)\bar{B}(z)^{-1}\right) . \tag{17}$$

From Property 3 in Theorem 1 the rank of the above matrix at $(x,z) = (0,0)$ is equal to n, so that $\frac{\partial\phi}{\partial x}(0,0)$ is invertible. From Property 2 in Theorem 1,

$$\forall (x,z) \in \mathbb{R}^{n+\bar{n}}\ ,\ \forall \lambda > 0\,, \quad \phi(\Delta_\lambda^{\tilde{r}}(x,z)) = \Delta_\lambda^r \phi(x,z)\,.$$

Let us now prove the following equivalence:

$$\phi(x,z) = 0 \Longleftrightarrow x = \psi(z) \qquad \text{with } \psi \in C^\infty(\mathbb{R}^{\bar{n}};\mathbb{R}^n)\,. \tag{18}$$

By application of the implicit function theorem, this property holds locally since $\phi(0,0) = 0$ and $\frac{\partial\phi}{\partial x}(0,0)$ is invertible. More precisely, there exists $\alpha > 0$ and a mapping $\psi \in C^\infty(B_{\bar{n}}(0,\alpha);\mathbb{R}^n)$ such that (18) holds whenever $z \in B_{\bar{n}}(0,\alpha)$. Since $z \in B_{\bar{n}}(0,\alpha)$ implies that $\Delta_\lambda^{\bar{r}} z \in B_{\bar{n}}(0,\alpha)$ when $\lambda \in [0,\lambda^*]$ ($\lambda^* > 0$), and since $\phi(x,z) = 0 \Longrightarrow \phi(\Delta_\lambda^{\tilde{r}}(x,z)) = 0$, one deduces that

$$\forall z \in B(0,\alpha),\ \forall \lambda \in [0,\lambda^*], \quad \Delta_\lambda^r \psi(z) = \psi(\Delta_\lambda^{\bar{r}} z)\,.$$

This shows that each component ψ_i of ψ is a smooth homogeneous function, thus polynomial. Therefore its domain of definition can be extended to $\mathbb{R}^{\bar{n}}$. Now, take any $(x,z) \in \mathbb{R}^{n+\bar{n}}$ such that $\phi(x,z) = 0$. There exists $\lambda \geq 0$ such that $\Delta_\lambda^{\bar{r}} z \in B_{\bar{n}}(0,\alpha)$. Since $\phi(\Delta_\lambda^{\tilde{r}}(x,z)) = 0$, one deduces that $\Delta_\lambda^r x = \psi(\Delta_\lambda^{\bar{r}} z) = \Delta_\lambda^r \psi(z)$ and thus $x = \psi(z)$. Therefore the $\Longrightarrow$ part of (18) is proved. As for the other side of the implication, if $x = \psi(z)$, then there exists $\lambda > 0$ such that $\Delta_\lambda^{\bar{r}} z \in B_{\bar{n}}(0,\alpha)$ and $\Delta_\lambda^r x = \psi(\Delta_\lambda^{\bar{r}} z)$. Therefore $\phi(\Delta_\lambda^{\tilde{r}}(x,z)) = 0$, which in turn implies that $\phi(x,z) = 0$.

From (18),

$$\frac{\partial\psi}{\partial z}(z) = -\left(\frac{\partial\phi}{\partial x}(\psi(z),z)\right)^{-1} \frac{\partial\phi}{\partial z}(\psi(z),z) = B(\psi(z))\bar{B}(z)^{-1}\,. \tag{19}$$

where the last equality comes from (16). Since $B(0)$ is of rank n, it follows that $\frac{\partial\psi}{\partial z}(0)$ is also of rank n. There subsequently exists a permutation matrix P such that

$$\frac{\partial\psi}{\partial z}(0) = (C_1\ C_2)\,P$$

with $C_1 \in \mathbb{R}^{n\times n}$ invertible. Let us now consider the mapping $\bar{\psi} \in C^\infty(\mathbb{R}^{\bar{n}};\mathbb{R}^{\bar{n}})$ defined by

$$\bar{\psi}(z) \triangleq \begin{pmatrix} \psi(z) \\ (Pz)_2 \end{pmatrix}$$

with $(Pz)_2 \triangleq \left(0_{(\bar{n}-n)\times n}\ I_{\bar{n}-n}\right) Pz$. We have

$$\frac{\partial\bar{\psi}}{\partial z}(0) = \begin{pmatrix} C_1 & C_2 \\ 0 & I_{\bar{n}-n} \end{pmatrix} P\,.$$

Let Δ'_λ denote the dilation on $\mathbb{R}^{\bar{n}-n}$ defined by $(P\Delta_\lambda^{\bar{r}} z)_2 = \Delta'_\lambda (Pz)_2$ and Δ''_λ the dilation on $\mathbb{R}^{\bar{n}}$ defined by

$$\Delta''_\lambda z \triangleq \begin{pmatrix} \Delta_\lambda^r & 0 \\ 0 & \Delta'_\lambda \end{pmatrix} z\,.$$

By using the fact that $\psi(\Delta_\lambda^{\bar{r}} z) = \Delta_\lambda^r \psi(z)$ one obtains $\bar{\psi}(\Delta_\lambda^{\bar{r}} z) = \Delta''_\lambda \bar{\psi}(z)$. This relation of homogeneity, combined with the fact that $\frac{\partial \bar{\psi}}{\partial z}(0)$ is invertible, implies that $\bar{\psi}$ is a diffeomorphism from $\mathbb{R}^{\bar{n}}$ to $\mathbb{R}^{\bar{n}}$. From (19), it is easy to show that, via this diffeomorphism, $\mathcal{F}_{\bar{n}}$ is transformed into the equivalent control system $\mathcal{S}^e$ given by (3) with $g_i(x,y) \triangleq \left(0_{(\bar{n}-n)\times n}\ I_{\bar{n}-n}\right) P\bar{b}_i(\bar{\psi}^{-1}(x,y))$. ∎

Proof of Lemma 3

Consider the following distribution at an arbitrary point $(x,y) \in \mathbb{R}^{\bar{n}} \times \mathbb{R}^{\bar{n}}$

$$\mathrm{Span}\left\{ \begin{pmatrix} \bar{b}_1(x) \\ \bar{b}_1(x) - \bar{b}_1(x-y) \end{pmatrix}, \ldots, \begin{pmatrix} \bar{b}_n(x) \\ \bar{b}_n(x) - \bar{b}_n(x-y) \end{pmatrix} \right\}.$$

Using assumptions b) and c) and the fact that, for any (i,j),

$$\left[\begin{pmatrix} \bar{b}_i(x) \\ \bar{b}_i(x) - \bar{b}_i(x-y) \end{pmatrix}, \begin{pmatrix} \bar{b}_j(x) \\ \bar{b}_j(x) - \bar{b}_j(x-y) \end{pmatrix} \right] = \begin{pmatrix} [\bar{b}_i, \bar{b}_j](x) \\ [\bar{b}_i, \bar{b}_j](x) - [\bar{b}_i, \bar{b}_j](x-y) \end{pmatrix}$$

one can verify that this distribution is involutive and of dimension equal to $\bar{n}$. The v.f. associated with this distribution are $\Delta_\lambda^{(\bar{r},\bar{r})}$-homogeneous. By application of Theorem 1 there exists a mapping $\phi \in C^\infty(\mathbb{R}^{\bar{n}} \times \mathbb{R}^{\bar{n}}; \mathbb{R}^{\bar{n}})$ such that $\phi(0,0) = 0$ and Property 2 of the lemma holds. Moreover, the rank of the jacobian matrix $(\frac{\partial \phi}{\partial x}(0,0)\ \frac{\partial \phi}{\partial y}(0,0))$ is equal to $\bar{n}$. By setting $y = 0$ in relation (6), one deduces that

$$\forall x \in \mathbb{R}^{\bar{n}}\,, \quad \frac{\partial \phi}{\partial x}(x,0)B(x) = 0\,, \quad \bar{B}(x) \triangleq \left(\bar{b}_1(x) \ldots \bar{b}_{\bar{n}}(x)\right)$$

and, since $\bar{B}(x)$ is invertible —by assumption b) and by homogeneity of the $\bar{b}_i$'s—,

$$\forall x \in \mathbb{R}^{\bar{n}}\,, \quad \frac{\partial \phi}{\partial x}(x,0) = 0\,.$$

Therefore, $\frac{\partial \phi}{\partial y}(0,0)$ must be invertible which shows that the sub-vector $\bar{x}$ in Theorem 1 is just a permutation of the components of y, i.e. $\bar{x} = (y_{\sigma(1)}, \ldots, y_{\sigma(\bar{n})})$. As a matter of fact, we can assume that $\bar{x} = y$ by changing ϕ into ϕ' defined by $\phi'_i = \phi_{\sigma^{-1}(i)}$. Therefore both Properties 1 and 3 of the lemma follow by application of Theorem 1. Now, since $\phi(0,0) = 0$ and $\frac{\partial \phi}{\partial x}(x,0) = 0$ —$\forall x \in \mathbb{R}^{\bar{n}}$—, one deduces that

$$\forall x \in \mathbb{R}^{\bar{n}}\,, \quad \phi(x,0) = 0\,.$$

This shows that the $\Longleftarrow$ part of Property 4 holds. Taylor's expansion of $\phi(x,y)$ in the neighborhood of $(0,0)$ yields

$$\begin{aligned}\phi(x,y) &= \frac{\partial\phi}{\partial y}(0,0)y + o(|y|) + O(|x||y|) \\ &= y + o(|y|) + O(|x||y|)\ .\end{aligned}$$

Therefore

$$\exists\epsilon > 0 : \quad (|x,y| < \epsilon \text{ and } y \neq 0) \Longrightarrow \phi(x,y) \neq 0\,. \tag{20}$$

Let us assume that there exists $(x^*, y^* \neq 0)$ such that $\phi(x^*, y^*) = 0$ then, by using Property 1,

$$\forall\lambda, \quad \phi(\Delta_\lambda^{\bar{r}} x^*, \Delta_\lambda^{\bar{r}} y^*) = 0$$

which contradicts (20). This proves the $\Longrightarrow$ part of Property 4. ∎

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